The Ensemble Square Root Filter (ENKF) [1] is one of the most widely used data assimilation method in a number of applications, including space weather. 2.1 ENKF Formulation For a vector of observations $y = \mathbf{r}$ and an ensemble of $N$ forecast $\mathbf{x}_i = \mathbf{r} + \mathbf{e}_i$, the ENKF analysis equation are given by:

$$
\mathbf{x}^a = \mathbf{x} + \mathbf{K} (\mathbf{r} - \mathbf{H} \mathbf{x})
$$

where $\mathbf{K} = \sigma^2 \mathbf{H}^T (\mathbf{R} + \mathbf{H} \sigma^2 \mathbf{H}^T)^{-1}$.

In the ENKF the forecast error covariance matrix is approximated through the model forecast, using the relation

$$
\mathbf{P}^f = \frac{1}{N} \sum (\mathbf{x}_i^f - \mathbf{x}^f) (\mathbf{x}_i^f - \mathbf{x}^f)^T
$$

where $\mathbf{x}^f$ is the $i^{th}$ forecast ensemble average.

2.2 Localized Adaptive Inflation To avoid filter divergence, due to lack of spread, the ensemble is inflated locally, using an adaptive localized inflation technique developed by Godinez and Koller [2] in the following way:

$$
\mathbf{x}^f = \mathbf{x}^a + \mathbf{K} (\mathbf{r} - \mathbf{H} \mathbf{x})
$$

where $\mathbf{K} = \sigma^2 \mathbf{H}^T (\mathbf{R} + \mathbf{H} \sigma^2 \mathbf{H}^T)^{-1}$.

2.3 Twin-Experiments Assimilation Results

3.1 One-Dimensional Experiment

**VERB** simulation setup:
- there is no diffusion across energy and pitch angle, only diffusion along the L-shells,
- given this simplification the assimilation only needs to be performed along the L-shells,
- assimilation experiment setup:
  - reference run is generated by including an artificial PSD source term to the initial condition of the L-shell solution following a sinusoidal path (open circles), which are assimilated into the ensemble from the control simulation (bottom plot in Figure 3). The overall structure of the solution towards the reference solution. Figure 2 shows the assimilation result from the twin-experiment, where the loss and energization of PSD is purely driven by the assimilation of the reference observations into the ensemble.

3.2 Three-Dimensional Experiment

**VERB** model is a three-dimensional radiation belt model developed at the University of California Los Angeles. The VERB model includes radial, pitch angle, and energy diffusion caused by ULF waves. The Versatile Electron Radiation Belt (VERB) model is a three-dimensional radiation belt model developed at the University of California Los Angeles. The VERB model includes radial, pitch angle, and energy diffusion caused by ULF waves. The Versatile Electron Radiation Belt (VERB) model. In particular, assimilation for the three-dimensional problem presents some challenges, as with the two-dimensional problem.

**Figure 4** shows the assimilation result for the three-dimensional experiment. As can be seen, the assimilation tries to correct the solution by nudging the control solution towards the reference solution, even though the reference solution has a time varying $K_p$ that ranges from 2 to 6 (dashed line in top plot of Figure 3) and the control solution has a constant $K_p = 2$. As with the one-dimensional result, the loss and energization are purely driven by data assimilation, demonstrating that the assimilation scheme works as expected.

4. Conclusions and Future Work

Conclusions:
- assimilation for the two-dimensional problem with ENKF problem provides a good solution for the estimated PSD, this problem only considers diffusion along the L-shell,
- assimilation for the three-dimensional problem provides reasonable results, although not as good as the two-dimensional problem,
- assimilation for 3D presents some challenges, although the diffusion of electrons along all spatial dimensions, this must be taken into consideration when computing the model covariance matrices.

Future Work:
- Develop a localization function that depends on diffusion coefficients used in model,
- assimilation of CRRES, RBSP, and other data sources into the three-dimensional model to enable reanalysis and predictions.

5. Acknowledgments

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References

Model and assimilation experiment setup:
- diffusion in L-shell, energy, and pitch angle,
- reference generated with time-varying $K_p$ control with constant $K_p = 2$,
- the assimilation uses 20 ensemble members, performed every 30 minutes, for a total time of assimilation of 5.0 days (240 assimilation cycles),
- the observations are taken over a sinusoidal path that ranges from 2 to 6 hours.

The loss experienced by the reference run in Figure 3 between days 3 and 4 is caused by an increase in $K_p$ on that day.

**Figure 1**: PSD plot for VERB two day simulation with $\mathbf{r} = \mathbf{r}_0 + \mathbf{W}_0$, $\mathbf{W}_0 \sim \mathcal{N}(0, \mathbf{R}_0)$, and $\mathbf{r}_0 = \mathbf{r}_T$, for 02:06:00. The top plot shows the reference run, using an artificial Gaussian source term placed at $L = 1.5$, and the bottom plot shows the control run without source term.

**Figure 2**: PSD as in Figure 1 for the assimilation result. Observations are taken from the reference solution following a sinusoidal path (open circles), which are assimilated into the ensemble. The assimilation result (analysis) is shown in the top plot, while the error is shown in the bottom plot. Figure 2 shows the assimilation result from the twin-experiment, where the loss and energization of PSD is purely driven by the assimilation of the reference observations into the ensemble.

**Figure 3**: PSD plot for VERB five day run with diffusion along L-shell, energy and pitch angle in the three-dimensional simulation. The top plot shows the reference PSD simulation with a time varying $K_p$, and the bottom plot shows the control PSD simulation with a constant $K_p = 2.0$. At both plots the dashed line indicate the plasma and the dotted line the $K_p$ value.

**Figure 4**: Analysis PSD from assimilating the reference solution (top plot in Figure 3) into an ensemble from the control simulation (bottom plot in Figure 3). The overall structure of the solution matches the reference simulation, indicating that the assimilation is nudging the control solution towards the reference solution.